PhD in Economics and Finance – 2022/2023
Bocconi University

Introduction to Probability

General presentation

The aim of the course is introducing in a mathematically rigorous language the measure-theoretical foundations of Probability. Furthermore, the following topics, used in Economics and Finance applications, are introduced:
Martingales, employed both in stochastic calculus and financial evaluation;
the probabilistic scheme for Bayesian inference;
M*arkov chains,* used both in simulation techniques and in stochastic modeling;
Stationary processes.
Timetable: six weeks; two lectures and one class per week.
Classes are intended to help the students in learning how to use the mathematical and probabilistic instruments. Exercises will be available to students both to verify the comprehension of the topics and to practice.

Contents

1 PROBABILITY SPACES AND RANDOM VARIABLES
Events and probability; Sequences of events; Random variables; Sigma-algebras and filtrations; Measurable random variables and random vectors; Approximation of a random variable by means of simple random variables; Independent sigma algebras
Independent random variables

2 EXPECTATION
Definition and properties; Limit theorems for expectations; Expectation and abstract integration; The L^2 space.

3 CONDITIONAL EXPECTATION AND MARTINGALES
General definition of conditional expectation; Properties; Conditional variance and variance decomposition; Definitions and examples of martingales; Stopping times; A convergence theorem for martingales.

4 CONDITIONAL PROBABILITY AND MARKOV CHAINS
General definition of conditional probability; Conditional distributions; The Bayesian scheme; Conjugate priors
Definition and properties of Markov chain; Transience and persistence; Stationary distribution; Convergence

5 LIMIT THEOREMS
Almost sure convergence and convergence in probability; Laws of large numbers; Convergence in distribution; The central limit theorem; Convergence of random vectors:

6 STATIONARY PROCESSES
Definitions; The Wold decomposition theorem; Laws of large numbers for serially dependent observations
Limit theorems for martingale differences.

Essential bibliography

Billingsley P. Probability and Measure Wiley
Grimmett G., Stirzaker D. Probability and Random Processes Oxford University Press
Karr A.F. Probability Springer - Verlag

Teaching aids

Lecture notes will be available before the starting of the course. Exercises will be given during the course.

Exam

Assignments (30%) and final written exam (70%).

Contact

For more information, please contact sandra.fortini@unibocconi.it