BOCCONI UNIVERSITY
DEPARTMENT OF FINANCE

PhD in ECONOMICS and FINANCE

CONTINUOUS TIME FINANCE

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Course Objectives

The course is an introduction to asset pricing in continuous-time. Firstly, we will introduce the necessary mathematical tools: Stochastic integration, (multi-dimensional) Ito's formula, and Girsanov's Theorem. For the classical Black-Scholes market, we will analyze the implications of the no-arbitrage assumption, the replication of contingent claims, and the concept of market completeness. We will extend the Black-Scholes model by considering time-dependent coefficients, stochastic volatility, jumps in the trajectories and several risky assets. Among other derivatives, American and path-dependent options will be priced.

Main References: Lecture notes will be distributed.
T.Bjork: *Arbitrage Theory in Continuous Time*, 4th Edition, Oxford University Press, 2020.
D.Duffie: *Dynamic Asset Pricing Theory*, Third Edition, Princeton University Press, 2001.

Syllabus

1. Stochastic processes in continuous time

1.1. Paths, information, measurability.

1.2. The Wiener Process.

1.3. The stochastic integral: The proper definition for financial applications.

1.4. Stochastic Differential Equations.

1.5. Ito processes. Ito formula.

2. The Black-Scholes model

2.1. The financial market and the securities.

2.2. Information and investment strategies. State prices and doubling strategies.

2.3. No-arbitrage, Equivalent Martingale Measures, Girsanov theorem and the market price of risk.

2.4. Completeness.

2.5. Pricing European derivatives.

2.6. Black-Scholes formula.

2.7. The Black-Scholes Partial Differential Equation. Examples.

2.8. American options: pricing and hedging.

3. *Financial markets with many risky assets*

3.1 The extension of the Black-Scholes model to many risky assets.

3.2 The multidimensional Ito formula and the multi-dimensional Girsanov theorem.

3.3 The Black-Scholes PDE for an European derivative on several underlying assets.

3.4 Exercises in the multidimensional market model.

3.5 State Price Densities. Hansen-Jagannathan bounds. The Growth Optimal Portfolio.

3.6 Currency markets and derivatives.

4. *The jump-diffusion model*

4.1 The jump-diffusion model.

4.2 The Ito formula for jump-diffusions.

4.3 No-arbitrage, Girsanov theorem for jump diffusions, completeness.

4.4 The formula for the European call option.

4.5 The infinitesimal generator for Ito processes and jump-diffusion processes.

5. The change of numeraire

5.1 Numeraires and related martingale measures. Examples in the diffusive setting.

5.2 Stochastic interest rates and forward-measures.

6. Portfolio and consumption choice

5.1 The Hamilton-Jacobi-Bellman equation.

5.2 Merton's Problem.

5.3 Solution to Merton's Problem via stochastic control.

7. Equilibrium

An example of equilibrium with power utility and infinite time horizon.

Assessment:

Presentation + take home

The list of topics will be distributed during the course. Students will present the selected topic in the last class of the course. A couple of related exercises to be solved as a take-home completes the assessment.