

Statistics for Extremes

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Course Objectives

Risk Analysis is a crucial research area that serves as a background to many scientific fields such as actuarial, financial, operational, engineering and environmental. The quantification of risks that a community, infrastructure or asset is exposed to as a consequence of adverse events, is of vital importance for governments and insurance companies to protect themselves from potential losses. The literature for risk analysis proposes a large amount of sophisticated methodologies to deal with this complex topic, both from the theoretical and quantitative point of view. One part of this literature is devoted to the study of rare or extreme events. The aim of this course is to provide students with a comprehensive overview of the basic approaches to modelling extreme events and a foundation for understanding the most recent research topics in this field.

In the course we introduce basic concepts of extreme or rare events. These events occur with low frequency but can have a large impact on real life. Therefore studying their stochastic behaviour is a crucial step for risk analysis. We discuss two basic models that are the cornerstones for modelling univariate extremes: the Generalised Extreme value and the Pareto distributions. We describe some basic techniques to estimate one-dimensional extreme events. Then, we introduce some approaches for modelling extreme events in high (finite) dimensions which are useful for describing multiple extreme observations. We describe different types of representations of the extremal dependence structure. Finally, we discuss some inferential methods to estimate high-dimensional extreme events.

Contents

1. Examples of limit distributions, partial maximum as sample extremes, three types of possible limit distributions, max-stable distributions. Exercises.
References: a) Ch. 3; b) Ch. 1; e) Ch. 2 Section 14 + Notes.
2. The class of extreme value distributions, domains of attraction, peaks-over-threshold (POT) and the Generalized Pareto (GP) distribution. Exercises.
References: a) Ch. 3-4; b) Ch. 1; c) Ch. 1; d) Ch. 2.2-2.3 + Notes.
3. Practical examples, how to use the theory for extrapolating extreme events. Exercises. b) Ch. 3 + Notes.
4. Introduction, max-id distributions, componentwise maxima as sample extremes, convergence of the dependence structure. Exercises.
References: a) Ch. 8; b) Ch. 6; c) Ch. 5 + Notes.
5. Characterization of the dependence structure, Poisson process construction, exponent measure, spectral decomposition, radial and angular measure. Exercises.
References: a) Ch. 8; b) Ch. 6; c) Ch. 5+ Notes.
6. Multivariate POT Approach and multivariate GP distribution.
References: d) Ch 5+ Notes.

Text Books

- a) S. G. Coles. *An Introduction to Statistical Modelling of Extreme Values*. Springer, London, 2001.
- b) L. de Haan and A. Ferreira. *Extreme Value Theory An Introduction*. Springer, USA, 2006.
- c) S. Resnick. *Extreme Values, Point Processes and Regular Variation*. Springer Verlag, New York, 1987.
- d) M. Falk, J. Hüsler and R-D. Reiss *Laws of Small Numbers: Extremes and Rare Events*, Birkhäuser, 2010.
- e) P. Billingsley. *Probability and Measure*. Wiley, 1995.

Assessment

Students have two options, take a written exam or alternatively to investigate a specific topic and make a presentation of it to the participants of the course.