Introduction to Mathematics

A.Y. 2023-2024 (Fall 2023)

Ph.D. in Economics and Finance, Bocconi University

Course Description:

- The course, which is a successor to the preparatory course, will continue covering the standard mathematical tools that are required for subsequent Ph.D. courses in Economics and Finance. We start with studying/reviewing set theory, linear algebra and basics of convex sets and functions from a higher standpoint. We move on to real analysis (topology) and convex analysis, with applications to economics and finance. Then, after studying/reviewing multi-variable calculus from a higher standpoint, we study (static) optimization and comparative statics of optimization problems. Finally, we briefly study dynamic programming. For some economic applications of the course materials and static and dynamic economic optimization problems, we will also study numerical implementation with MATLAB (see course outline below).

- The course will cover both abstract mathematics and applications to economics and finance. For core mathematical concepts whose conceptual or logical understanding is essential, the course materials (lectures and problem sets) are largely proof-based. In contrast, for certain topics when the emphasis of the course is for you to be able to use certain mathematical results correctly in applied settings, the focus during the course is to enable you to understand and use the results correctly as a user.

- The detailed syllabus (for example, with information on office hours) will be distributed at the course website.

Prerequisites: Undergraduate-level Calculus, Matrix/Linear Algebra, and Probability required for economics and finance

Instructor (*both* lectures and TA sessions):

- Satoshi Fukuda (Department of Decision Sciences)
- Lecture, TA, and Office Hours: TBA

Course Outline (Tentative):

1. Fundamentals of Set Theory
    - Sets; Basic Operations of Sets; Mappings; Relations; Methods of Proof
2. Fundamentals of Vector Spaces

- Vector Spaces; Linear Mappings; Vector Subspaces; Dimension of a Vector Space

3. Introduction to Convex Sets and Functions

   - Convex Sets; Convex/Concave Functions

4. Inner-Product, Normed, and Metric Spaces

   - Inner Product; Norm; Distance; Open and Closed Sets; Limits and Continuous Functions; Compact Sets; Unconstrained Optimization; Semi-continuous Functions; Completeness and Contraction Mapping Theorem; Representation, Projection, and Approximation; Applications of Projection Theorem (OLS, Normal Updating)
   - MATLAB: Unconstrained Optimization and Projection Theorem (OLS)

5. Convex Analysis on $\mathbb{R}^n$

   - Separation Theorems; Supporting Hyperplane Theorems; Minimax Theorem; Farkas-Minkowski Theorem

6. Differentiable Functions on $\mathbb{R}^n$

   - Differentiable Functions; Differentiable Convex/Concave Functions; Implicit Function Theorem

7. Optimization, Comparative Statics, and Value Functions

   - Kuhn-Tucker Theorem
   - Comparative Statics of Smooth Optimization Problems; Value Functions and Envelope Theorems
   - MATLAB: Constrained Optimization (Kuhn-Tucker Theorem)
   - Correspondences; Theorem of the Maximum

8. Introduction to Dynamic Programming

   - Bellman's Principle of Optimality
   - MATLAB: Simple Economic Examples

Course materials and textbooks: There are no required textbooks for the course. The course material will draw from the lecture notes to be available at the course website. For those who would like to consult textbooks in addition to the lecture materials, a list of textbooks will be indicated in the lecture notes.

Programming Languages: As indicated above, this course covers numerical optimization for (simple) static and dynamic economic problems. The programming language with which we will work during the lectures and TA sessions is MATLAB. However, for problem sets, I do not require the choice of a programming language. Those who would like to work with, for example, Python or Julia are also welcomed to do so.

Grading: A combination of problem sets (both theory and programming) and final exam. Precise information will be announced.