

40032 – Introduction to Real Analysis I

Instructor: Hugo Lavenant
Email: hugo.lavenant@unibocconi.it

Prerequisites: undergraduate real analysis (set theory, familiarity with $\varepsilon - \delta$ arguments, infinite series, basics of topology).

Course description. We will cover measure theory and integration. The course will hopefully be useful for students whose research will involve rigorous proofs in probability theory, statistics and functional analysis.

Course outline

1. σ -algebras, measurable functions and measures.
2. Integration.
3. Construction of measures. Lebesgue-Stieltjes measures on $\mathbb{R}$.
4. Product measures, Fubini's theorems.
5. Signed measures.
6. The Radon-Nykodim theorem.

Objectives. The goal is for the students to understand the concepts of measure theory and to be comfortable with abstract mathematical reasoning. In addition to the understanding of measure theory, the students will be evaluated on their ability to design and write mathematical proofs.

Lecture notes and textbooks. Lecture notes will be distributed at the beginning of the course. They are strongly inspired from the following lecture notes:

- J.-F. Le Gall, *Measure Theory, Probability, and Stochastic Processes*. Springer. (Chapters 1,2,3,5 and 6).

You can find material close to what we will cover in the following textbooks:

- R. B. Ash and C. Doléans-Dade, *Probability and Measure Theory*. Academic Press. (Chapter 1 and parts of Chapter 2).
- W. Rudin, *Real and Complex Analysis*. McGraw-Hill Higher Education. (Chapters 1,2,3,6 and 8).

Exercises. Exercises will be posted online. They are of varying difficulty and are quite similar to what will be asked during the final examination.

Examination. There will be a single final *written* examination.