Università Bocconi
DEPARTMENT OF DECISION SCIENCES

Mathematics & Music: Quantitative Emotions

Emanuele Borgonovo
Full Professor, Department of Decision Sciences, Bocconi University
Research Affiliate, Department of Nuclear Science and Engineering, MIT
emanuele.borgonovo[at]unibocconi.it

Abstract

Music is a universal language. It needs no translation, but it reaches the hearts and minds of all human beings. What are the secrets of its beauty? What makes it communicate emotions like joy and tranquility rather than sadness or rage? How do composers create the wonders we all listen to? This course will guide us on an in-depth tour of the inner mechanisms of music. We will explore the meaning of melody and the relationships between mathematics and harmony. We will start with harmonic ratios and progress through advanced chords in classical and jazz music. We will learn about the surprising circle of fifths and touch upon the concept of counterpoint. Then, we will examine some of the main musical structures, looking into the form of songs and sonatas. Surprisingly, we will see that mathematical concepts like periodicity, fractals, and topology have musical counterparts. We will consider how virtuosity and instrumentation help build emotions, listening through a limitless series of incredible pieces. We will also explore the connections between music and movies, music and paintings, music and computers. We will discuss the role of artificial intelligence in music, from the creation of playlists, to the creation of music itself by quantum computers. Throughout, we will examine how the scientific literature has addressed these topics in what Johann Sebastian Bach called "the Science of Music". A rich selection of listening examples and videos will accompany our journey.

Calendar

Session 1 **24 March 2026** 6:15-7:45PM Room TBA

Session 2 **31 March 2026** 6:15-7:45PM Room TBA

Session 3 **14 April 2026** 6:15-7:45PM Room TBA

Session 4 **16 April 2026** 6:15-7:45PM Room TBA

Session 5 **21 April 2026** 6:15-7:45PM Room TBA

Session 6 + FINAL ASSIGNMENT **23 April 2026** 6:15-7:45PM Room TBA

Detailed Syllabus

Here is an extended plan of our lectures. The lectures will be full of musical readings, videos, discussions and recordings to illustrate the contents below.

Module 1: Unveiling the Mystery of Harmony and Musical Scales

What creates a chord? And a Melody? We will illustrate how two elementary mathematical operations, ratios and additions, are at the basis of the beauty of musical creation.

1.1 The foundations: Sound and Mathematics

- Sound waves and pitches
- The harmonic series, Fourier analysis: decomposing sound into pure frequencies
- Greek ratios and consonance, Just intonation vs. equal temperament: a journey from Pythagoras to Bach

1.2 The Magic of Harmony: From Ratios to Chords and Tones

- Simple but powerful chords
- Harmonic progressions and tonal direction
- Dissonance and resolution
- Surprising harmonies in jazz
- Harmonic evolution in great composers

Module 2: Reading Musical Sentences

We will progress a bit more, creating together our own harmony of a known theme.

2.1 The Circle of Fifths and the Art of Modulation

(What is tonality, indeed?)

- Tonal centers and harmonic gravity
- Scriabin's idea of colors and music
- Modulation as structural transformation
- Cycles and symmetry in the circle of fifths
- Periodic harmonic modulations

2.2 Melody and Harmony

We will examine music as language and how it can be structured through the examples of great masters, from the past and the present.

- Musical phrases and breathing
- Vertical structure: notes forming chords
- Horizontal structure: melody moving between chords

- Passing tones, suspensions (*ritardi*), and appoggiature
- The beauty of controlled dissonances

Module 3: More Mathematics for More Music

In this section, we will search the ocean of music for wonderful examples, hidden gems and precious pearls.

3.1 The Magic of Polyphony

- The Counterpoint
- The Canon
- The Fugue
- Independence and interaction of voices
- Mathematical balance and symmetry
- Mind-blowing musical examples

Module 4: The Theoretical Thinking Behind Music and Complex Musical Architectures

4.1 Musical Theories

This section hints at the formal mathematical language behind harmonic transformations, in a scientifically oriented way.

- Hugo Riemann's harmonic theory and Riemannian transformations
- Tonal relations as transformations
- Group theory and musical structure
- The intuition besides musical spaces
- Geometry of harmony

4.2 Music and Symmetries

- Symmetry and transformations
- Fractals, self-similarity: Augmentatio and Diminutio
- Anti-symmetry and theme inversion
- Mind-Blowing Examples

4.3 Musical Structures

- Songs
- Sonatas
- Fugue
- Theme & Variations in classical music and Jazz
- Even more mind-blowing large-scale musical architectures

Module 5: Music and Emotions

5.1 Music and Rhythm: from Jazz to *Precipitato*

- Rhythmic patterns and syncopation
- Swing and groove in jazz
- Polyrhythms and cross-rhythms
- Irregular meters (5/4, 7/8, etc.)
- Rhythm as mathematical structure

5.2 Music and Movies: Timing, Structure, and Visual Rhythm

- Music as storytelling and emotional architecture
- Synchronisation: mathematics of timing
- Musical gesture and animated motion
- Expectation and surprise in musical soundtracks

5.3 Structure and Virtuosity

- How structure shapes emotional experience
- Tension and release as mathematical design
- Expectation and transformation
- Virtuosity as expressive intensity

5.4 Even more Emotions and Creativity

- Music and Paintings
- Music and Mathematicians
- Virtuosity as expressive intensity
- Smiling with Music

Module 6: Music and AI, when Algorithms help Creativity, and the Future

In this section, we will overview recent developments at the intersection of AI and music.

- Crating playlists: from juke boxes to Spotify
- AI Music Software
- Neural networks and style imitation
- Music from Quantum Computers

Course Materials

- Emanuele Borgonovo: Notes and Slides on Mathematics and Music.

Books

- Julian Hook, 2023: Exploring Musical Spaces, Oxford University Press.
- Paul Hindemith: The Craft of Musical Composition, translated by Artur Mendel, Associated Music Publishers Inc., New York.
- Arnold Schoenberg: Theory of Harmony, translated by R.E.Carter, University of California Press.
- Luigi Molfino: Lo Studio dell'Armonia, 2009, Edizioni Carrara.
- Joseph Fux, 1725: Gradus at Parnassum (several editions and translations)
- Theodore Dubois, 1996: Treatise on Counterpoint, Ricordi.

Scientific Articles

- Callender, C., Quinn, I., and Tymoczko, D. (2008). Generalized voice-leading spaces. Science, 320(5874), 346–348. https://doi.org/10.1126/science.1153021
- Tymoczko, D. (2006). The geometry of musical chords. Science, 313(5783), 72–74. https://doi.org/10.1126/science.1126287
- Hook, J., 2002: Uniform Triadic Transformations, Journal of Music Theory, 36 (1/2), pp. 57-126.